2D Control Problem and TVD-Particle Method for Water Treatment System

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Analysis of the environmental impact of waste water discharges into aquatic media takes a great importance in the last years. The problem considered here is in the field water-quality improvement by varying the systems, such as flow regulation by means of reservoirs. The criterion functional to be minimized penalizes deviation of fecal coliform distribution from standard value. We address approximate controllability problems for a parabolic equation (evolution of pollutant concentration) associated with Dirac measures. From a practical point of view, a regularized version of the problem is considered. The simulation of the transport and fate of compounds within biological systems are done by an hybrid numerical approach combining particle method and finite difference technique. Symmetric TVD scheme [1] for the shallow water equations is provided to minimize numerical diffusion. The technique is a composed method that uses a second order flux in smooth regions but involves some limiting based on the gradient of the solutions so that near discontinuities it reduces to the monotone upwind method. A particle method is proposed to handle the parabolic equation. The difficulty is then to deal with a diffusion term and boundary conditions. The approximation of the diffusion operator is based on the introduction of boundary integral equation formulation [2]. In the minimizing algorithm, the gradient of the cost function is evaluated by adjoint techniques and a gradient type method as an iterative solution of the discrete control problem is chosen. The major issue of the numerical part relies on illustrating by direct simulation the effectiveness of this methodology when applied to the control of water quality problem. Relative merits and advantages of this approach are explored. The purpose of this talk is to present recent results which generalize those obtained in [1], [2] in the 2D case.

References

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[2] M. Louaked, A. Saidi (2009) Numerical Aspects of Optimal Control for Water Treatment Problem, in *Systems Theory: Modeling, Analysis and Control, Presses Universitaires de Perpignan* pp. 335–342.

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