The Concepts of Polynomial Orthogonality and Their Applications

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Since the orthogonal polynomials have a lot of applications in mathematics (theory of special functions, fractional calculus, spectral operators, probability, analytic numbers and approximations) and other sciences (physics, mechanics, electrotechnics), they appeared numerous extensions of the concept of the standard orthogonality. We will discuss the concepts of quasi, almost and multiple orthogonality comparing their similarities and differences in: recurrence relations, differential properties, generating functions, Rodrigues formulas and zeros. As some of their most important applications, we will consider quadratures and approximations of functions. Especially, we will discuss almost orthogonal polynomials which are less known, but very useful in modeling of electronic systems which generate orthonormal basis. They are very suitable for analysis and synthesis of imperfect technical systems which are projected to generate orthogonal polynomials, but in the reality generate almost orthogonal polynomials.

As a next step, we plan to consider the relations of these new orthogonality's concepts to the operators of the fractional calculus, based on fractional order generalizations of the Rodrigues differential (or differintegral) formulas for the classical orthogonal polynomials.

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