Mathematical Models in Dynamics of Incompressible Viscoelastic Media

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A consistent model of incompressible viscoelastic Maxwell media is formulated. It corresponds to the choice of Jaumann rotational derivative in rheological relation. The determining system of equations has both real and complex characteristics. For this system, a solvability of initial-boundary value problem in the class of analytic functions is established, for its linearized variant the solvability was shown in the class of functions of finite smoothness. It was shown that the absence of short-wave instability was provided by smallness of non-diagonal terms of stress tensor. A wide class of exact solutions to motion of incompressible viscoelastic Maxwell medium is found. These solutions are partially invariant with respect to some sub-group of extended Galilei group which is admitted by equations of motion and their generalizations. With the help of obtained solutions there was described a deformation of viscoelastic strip with free boundaries, which moves either inertial or under the action of stretching or compressing longitudinal stresses, as well as shear stresses, applied to the free surface. A problem on spherical cavity filling in incompressible Maxwell medium under the action of constant pressure at infinity was considered. This is generalization of the classic problem for viscous incompressible liquid. In both cases cavity always converges to a point in a finite or infinite time. In case the surface tension differs from zero, the collapse happens in a finite time. Depending on the three dimensionless parameters (Reynolds number, capillary number and dimensionless relaxation time) both oscillatory and monotonic regimes of motion are possible. When the cavity radius becomes small, oscillations stop.

There was also considered a problem on spherical cavity filling in incompressible viscoelastic Kelvin-Voigt medium under action of constant pressure at infinity. On the contrary to similar problem for Maxwell medium, here both the cavity collapse and stabilization of its radius to a positive value with time are possible. It was shown that finiteness of deformations leads to new qualitative effects. There were revealed three different modes of boundary behavior, two of which stand for monotonous and oscillatory regimes of limit cavity radius achieving, while the third one stands for cavity collapse in a finite time.

Classical Couette problem analogues of incompressible viscoelastic Kelvin-Voigt and Maxwell media motion in the gap between two coaxial cylinders are investigated. External cylinder is fixed, while the internal one is either forcibly rotating with a constant velocity or inertially rotating. There was found analogy between propagation of acoustic cylindrical waves in viscous gas and transversal waves in Kelvin-Voigt medium which appear during its motion along circular trajectories. Asymptotes of solution stabilization to Couette motion in the first problem and inertial cylinder motion approaching to equilibrium state were studied. In the second problem for Maxwell motion there are also studied limit regimes of rotation retardation. In comparison to Kelvin-Voigt medium where both monotonous and oscillatory regimes of rest state approaching are possible, for Maxwell medium the velocity of rotation with time tends to zero in oscillatory regime.

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