Mathematical Models and Distribution of the Zeros of the Duffing Equation and More General Equations

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Here we investigate the oscillation behavior of the equation:

$$\ddot{x}(t) + \delta\dot{x}(t) + \sum_{k=1}^{n} \beta_k(x(t-\sigma_k), t) + G(\ddot{x}(t), \dot{x}(t), x(t)) = f(t),$$

generalizing two recent results of Petrova. The Duffing equation is:

$$\ddot{x}(t) + \delta\dot{x}(t) + \alpha x(t) + \beta x^3(t) = f(t).$$

The common assumptions are that $\alpha > 0$, $\beta > 0$ and $\delta \in \mathbf{R}$ are constants and $f(t) \in C([T, \infty); \mathbf{R})$, T is a large enough constant. Also, in the general case we suppose that all the delays $\{\sigma_k\}_{k=1}^{n}$ are nonnegative constants as well as that $G(\ddot{x}(t), \dot{x}(t), x(t)) \in C(\mathbf{R}^3; \mathbf{R})$ and $\beta_k(x(t-\sigma_k), t) \in C(\mathbf{R}; \mathbf{R})$, $\forall\ k = 1 - n$, $n \in \mathbf{N}$.

Further, we make several approaches, concerned with the mathematical intuition based on the distribution of the zeros of wide classes of second order ordinary differential equations. The first one treats the bifurcation theory. The second one is connected with proper numerical methods.

Keywords: oscillation, the Duffing equation, limit cycle, numerical methods

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