Convergence of Series in Mittag-Leffler Type Functions

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The Mittag-Leffler (M-L) functions are natural extensions of the exponential function and trigonometric functions like cos-function. Their basic properties are described yet in the Bateman Project "Higher Transcendental Functions" (Vol. 3), in a chapter devoted to "miscellaneous functions." The functions have been studied in details by Dzrbashjan [1]. The detailed properties of these functions can be found in the contemporary monographs [3], [4], [7]. The same functions, considered also in [5] and [8], are called M-L functions of vector index. Recently a class of special functions of M-L type, that are multi-index ana- logues of M-L functions, has been introduced and studied (see, e.g., [4]). Explicit solutions of some kinds of fractional order (or multi-order) differential and integral equations involving Erdelyi-Kober (E-K) operators are representable by means of series in M-L type functions like those considered in this work. Their domains of convergence are found. The series behaviour are studied on the boundary of these domains. Cauchy-Hadamard, Abel, Tauber and Littlewood type theorems for such series are given. Asymptotic formulae for "large" values of indices of these functions are also provided, used in the proofs of the convergence theorems for the considered series. In our previous papers [6] we studied series in systems of some representatives of special functions of fractional calculus (SF of FC) which are fractional indices analogues of the Bessel functions and also multi-index M-L functions (in the sense of Kiryakova [4]).

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