Shallow Water Model Which Admits the Propagation of Shocks over a Dry Bed

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The equations of the first approximation of shallow water theory are widely used to model the propagation of hydraulic bores generated by total or partial dam breaks or by the impact of large sea tsunami-type waves on shallow beaches. The theory describes such bores as steady discontinuous solutions, which we will name shocks. However, the classical system of the basic shallow water conservation laws (which consist of the conservation laws of mass and total momentum) while correctly describing the parameters of hydraulic bores propagating in a fluid of finite depth, does not admit shock propagation over a dry bed. Exact solutions describing dry-bed water flows governed by this system, for example, solution of a dam break problem with a dry bed in the tail water, are continuous depression waves. At the same time, numerous laboratory experiments have shown that wave front velocity in continuous solutions is considerably overestimated and the wave front profile noticeably distorted. Experiments have shown that the wave fronts propagating over a dry bed are much steeper and exhibit breakdowns characteristic of hydraulic bores.

In the present work, a method for modeling the propagation of discontinuous waves over a dry bed using the first approximation of shallow water theory is proposed. The method is based on a modified conservation law of total momentum that takes into account the concentrated momentum losses due to the formation of local turbulent vortex structures in the fluid surface layer at a discontinuous-wave front. A quantitative estimate of these losses is obtained by deriving the shallow water equations from the Navier-Stokes equations with allowance for viscosity, which has a rapidly increasing effect in the turbulent flow regions described by discontinuous waves. As an example, a comparative analysis is performed of the solutions of the dam-break problem obtained for the classical and modified shallow water models.

A generalization of modified shallow water model for two dimensional case with variable bottom height is proposed. It is presented the numerical results obtained for the propagation over a dry bed of two dimensional shock caused by a partial dam break in a channel with variable bottom in the lower pool.

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