Total Robustness of Nonconforming Finite Element Methods

S. Margenov

Institute for Parallel Processing of Information, Bulgarian Academy of Sciences
25 Acad.G. Bonchev str., 1113 Sofia, Bulgaria

The talk is in the field of large-scale scientific computing. The considered ill conditioned problems are described by partial differential equations (PDEs) with small parameters. The finite element method (FEM) is used for discretization. The total robustness means: a) robust finite element approximation; and b) robust solution of the arising FEM linear algebraic systems.

It is well known that the standard (conforming) finite elements don't provide stable approximation for certain ill conditioned problems. Here we consider the Crouzeix-Raviart (C-R) nonconforming elements as an alternative in such cases. The robustness of the C-R elements for elliptic problems in strongly heterogeneous media is explained by their equivalence to the mixed FEM discretization with lowest order Raviart-Thomas elements. We present also robust applications of C-R FEM for the following two strongly coupled ill conditioned PDEs: a) the Lame system of linear elasticity in the case of almost incompressible materials; and b) the time dependent equations of Navier-Stokes in the case of large Reynolds number.

The second part of the talk deals with the robustness of multilevel iterative solvers for the linear systems arising from the above described nonconforming FEM discretizations. The presented results are in the spirit of optimal order Algebraic MultiLevel Iteration (AMLI) methods (see, e.g., [1]). The theory of the AMLI methods is based on hierarchical decomposition of the FEM spaces where the constant in the strengthened Cauchy-Bunyakowski-Schwarz (CBS) inequality takes a particular role. Uniform estimates of the CBS constant are shown for the corresponding elliptic and parabolic problems. In addition to the problem related small parameters, the AMLI methods are robust with respect to mesh anisotropy.

The theoretical results are illustrated by a rich set of numerical tests.

References

[1] J. Kraus, S. Margenov, *Robust Algebraic Multilevel Methods and Algorithms*, de Gruyter, 2009.

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