Finite Difference Schemes for Multidimensional Boussinesq Equation

N. Kolkovska

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences
Sofia, Bulgaria

We consider the Cauchy problem for the nonlinear Boussinesq equation

$$\begin{aligned}\frac{\partial 2u}{\partial u^2} =& \Delta u + \beta_1 \Delta \frac{\partial^2 u}{\partial t^2} - \beta_2 \Delta^2 u + \alpha \Delta f(u), \quad x \in \mathbb{R}^d, \quad t > 0, \\ u(x,0) = u_0(x),& \qquad \frac{\partial u}{\partial t}(x,0) = u_1(x),\end{aligned}$$

where α, β_1, β_2 are positive constants and the solution u additionally satisfies the asymptotic boundary conditions $u(x,t) \to 0$, $\Delta u(x,t) \to 0$ as $|x| \to \infty$. Typically, the nonlinear term is $f(u) = u^2$.

For the numerical solution of this equation families of *conservative finite difference schemes* are constructed and studied theoretically.

Depending on the way the nonlinear term $f(u)$ is approximated, we develop iterative and non-iterative schemes, which are second order accurate in space and time. Our numerical experiments show clear advantage of the non-iterative schemes in precision and speed.

The schemes are implemented in fast, memory efficient and numerically stable methods. The extensive numerical experiments show good precision in real time computations and full agreement between the theoretical results and practical evaluation for single soliton and the interaction between two solitons.

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