Accelerated Iterations for Finding the Soft-Constrained Stochastic Nash Games Equilibrium

I. G. Ivanov, R. Rusinova, and B. Lomev

Dept of Statistics and Econometrics, Faculty of Economics and Business Administration St. Kl. Ohridski University of Sofia, 125 Tzarigradsko chaussee Blvd., bl. 3, Sofia 1113, Bulgaria

The following constrained weakly coupled system is considered [1]:

$$\begin{aligned} X_1(A_0 - S_2X_2) + (A_0 - S_2X_2)X_1 + A_1^T X_1 A_1 - X_1 S_1 X_1 + X_1 M_1 X_1 + \epsilon X_2 S_{12} X_2 + Q_1 = 0 \\ X_2(A_0 - S_1X_1) + (A_0 - S_1X_1)X_1 + A_1^T X_2 A_1 - X_2 S_2 X_2 + X_2 M_2 X_2 + \epsilon X_1 S_{21} X_1 + Q_2 = 0 \end{aligned} \tag{1}$$

with the $m \times m$ unknown symmetric matrices X_1, X_2 is called a set of cross-coupled stochastic algebraic Riccati equations (CSARE) under notations

$$\begin{aligned} S_i = B_i R_{ii}^{-1} B_i^T, \qquad M_i = E V_i^{-1} E^T, \qquad i = 1, 2, \\ S_{21} = B_1 R_{11}^{-1} R_{21} R_{11}^{-1} B_1^T, \qquad S_{12} = B_2 R_{22}^{-1} R_{12} R_{22}^{-1} B_2^T. \end{aligned}$$

Mukaidani [1] has proposed the following stochastic algebraic Lyapunov iteration:

$$\begin{aligned} X_1^{(k+1)} \tilde{A}_{1,k} + \tilde{A}_{1,k} X_1^{(k+1)} + A_1^T X_1^{(k+1)} A_1 + X_1^{(k)} S_1 X_1^{(k)} - X_1^{(k)} M_1 X_1^{(k)} + \epsilon X_2^{(k)} S_{12} X_2^{(k)} + Q_1 = 0 \\ X_2^{(k+1)} \tilde{A}_{2,k} + \tilde{A}_{2,k} X_2^{(k+1)} + A_1^T X_2^{(k+1)} A_1 + X_2^{(k)} S_2 X_2^{(k)} - X_2^{(k)} M_2 X_2^{(k)} + \epsilon X_1^{(k)} S_{21} X_1^{(k)} + Q_2 = 0 \end{aligned}$$

with $\tilde{A}_{i,k} = A_0 + S_1 X_1^{(k)} + S_2 X_2^{(k)} + M_i X_i^{(k)}$ for finding a positive semidefinite solution to (1).

We propose a new iteration scheme for solving (1) and we compare a few iterations for computing a positive definite solution to (1). We will compare the numerical effectiveness of the corresponding recursive procedures. Numerical examples are used to demonstrate the performance.

Acknowledgements. This paper was financially supported under the St. Kl. Ohridski University of Sofia research project 2010.

References

[1] H. Mukaidani (2008), Infinite-Horizon Soft-constrained Stochastic Nash Games with State-dependent Noise in Weakly Coupled Large-Scale Systems, 2008 American Control Conference, USA, pp. 4232-4237.

$\rightarrow \infty \diamond \infty \leftarrow$