On Soliton Equations and Soliton Interactions

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I will describe an important class of nonlinear evolution equations in two-dimensional space-time that are exactly integrable and find applications in nonlinear optics, hydrodynamics, plasma physics, superconductivity etc. Two important representatives of the soliton equations is the nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + |u(x,t)|^2 u(x,t) = 0.$$

and the sine-Gordon equation (sGE):

$$\frac{\partial^2 v}{\partial x \partial t} = \sin v(x,t).$$

I will first outline the inverse scattering method that allows one to solve the NLSE and the sGE, and how to obtain explicitly their N-soliton solutions. Due to their stability they allow one to explain effects that can not be obtained by perturbation theory.

Then I will explain the soliton interactions and the effects of perturbations on them.

Finally I will address the generalizations of this theory to multicomponent NLSE related to symmetric spaces. Special attention will be paid to the system

$$\begin{aligned}
&i\partial_t \Phi_1 + \partial_x^2 \Phi_1 + 2(|\Phi_1|^2 + 2|\Phi_0|^2)\Phi_1 + 2\Phi_{-1}^* \Phi_0^2 = 0,\\
&i\partial_t \Phi_0 + \partial_x^2 \Phi_0 + 2(|\Phi_{-1}|^2 + |\Phi_0|^2 + |\Phi_1|^2)\Phi_0 + 2\Phi_0^* \Phi_1 \Phi_{-1} = 0,\\
&i\partial_t \Phi_{-1} + \partial_x^2 \Phi_{-1} + 2(|\Phi_{-1}|^2 + 2|\Phi_0|^2)\Phi_{-1} + 2\Phi_1^* \Phi_0^2 = 0.
\end{aligned}$$

which describes Bose-Einstein condensate of alcali atoms in the $F = 1$ hyperfine state and elongated in one direction.

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