Methods for the Coupled Stokes-Darcy Problem

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The motion of particles in a viscous fluid close to a porous membrane is relevant for microfiltration applications. Here, particles are assumed to be large compared with the size of pores of the membrane. The hydrodynamic interactions of one particle with the membrane are considered here. The model involves Stokes equations for the fluid motion around the particle together with Darcy equations for the flow in the porous membrane and Stokes equations for the flow on the other side of the membrane. Boundary conditions at the fluid-membrane interface are the continuity of pressure and velocity in the normal direction and the Beavers & Joseph slip condition on the fluid side in the tangential directions. The no-slip condition applies on the particle. This problem is solved here by two different methods.

The first one is an extended boundary integral technique. A Green function is derived for the flow close to a porous membrane. This function is non-symmetric, leading to difficulties hindering the application of the classical boundary integral technique. Thus, an extended technique is proposed, in which the unknown distribution of singularities on the particle surface is not the stress, like in the classical boundary integral technique. Yet, the hydrodynamic force and torque on the particle are obtained by integrals of this distribution on the sphere surface.

The second method consists in searching the solution as an asymptotic expansion in term of a small parameter that is the ratio of the typical pore size to the particle size. The various boundary conditions are taken into account at successive orders: order (0) simply represents an impermeable wall without slip and order (1) an impermeable wall with a peculiar slip prescribed by Order (0); at least the 4th order is necessary to enforce all boundary conditions.

The methods are applied numerically to a spherical particle and comparisons are made with earlier works in particular cases.

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