

# A Note on Interpolation Functions of the Frobenius-Euler Numbers

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The main purpose of this paper is to construct partial twisted zeta-type function, which interpolates  $q$ -generalized Frobenius-Euler numbers and twisted  $q$ -generalized Frobenius-Euler numbers. We give some applications related to this function.

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