Stochastic Super-exponential Growth Model Describing Population Dynamics

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Observational data clearly suggest that the human population has grown faster than exponentially, exhibiting a super-exponential growth for most of the known history. The accelerating character of the growth rate describing the human population can be generalized as an inherent property of evolutionary processes. Based on the analogy with different evolutionary processes we have concluded that the human population is a dynamical system with non-linear positive feedback occurring between technological advancement and the abundance of a population. Behavior of a super-exponential function asserts that a population, growing at a super-exponential pace, will encounter a catastrophe within a finite time. This catastrophic event is mathematically described as a singular state, where population has exploded to infinity. In current paper the influence of the environmental noise to superexponentially growing population is considered. Ascribing stochastic nature to Malthusian parameter r, which denotes an influence of the environmental noise to a growth of a population, we consider a model that follows from the equation

$$\frac{dy}{dt} = -\alpha r + \sqrt{D}\zeta(t) \tag{1}$$

where t is time, α is a parameter describing the technological advancement of the population, D is the intensity of the Gaussian white noise $\sqrt{D}\zeta(t)$, interpreted in the sense of Ito. In equation (1) $y = p^{-\alpha}$, where p denotes the abundance (number of members or density) of a population. The corresponding Fokker-Planck equation is solved. The mean value and dispersion of the population density p as well as the average lifetime of the system together with its dispersion are calculated. Implications of the mathematical analysis of the stochastic super-exponential model on the biological system have also been examined. Various applications of the results of the analysis are discussed.

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