Analysis of Types of Oscillations in Goodwin's Model of Business Cycle

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We study numerically the properties of nonlinear oscillations of Goodwin's model of business cycle [1]

$$\begin{cases} \epsilon\dot{y}(t) + (1-\alpha)y(t) = \varphi(\dot{y}(t-\theta)) + l(t) + \beta(t), & t > 0 \\ y(t) = \phi(t), & -\theta \le t \le 0. \end{cases}$$

Here $y(t)$ is income, $\epsilon^{-1} > 0$ – the adjustment coefficient, α – the marginal propensity to consume, $0 \le \alpha \le 1$, $\phi(\dot{y}(t))$ – the induced investment function, θ is a lag in the investment function, $l(t)$ is autonomous investment, and $\beta(t)$ is autonomous consumption,

$$\phi(z) = \begin{cases} \phi_+, & z > \kappa_1\phi_+ \\ \kappa z, & \kappa_1\phi_- \le z \le \kappa_1\phi_+ \\ \phi_-, & z < \kappa_1\phi_- \end{cases}$$

κ is the acceleration coefficient, ϕ_- and ϕ_+ are upper and lower limits of ϕ.

In simulating this model on an analog computer [2] was indicated the existence of Goodwin's limit cycle solution with a period much greater than θ and an infinite number of limit cycle solutions with periods θ, $\theta/2$, $\theta/3$,... . Our calculations show that such cycles may be caused by high-frequency oscillations of the initial function $d\phi(t)/dt$. We have shown also that Goodwin's limit cycle solutions exist only in a limited range of θ, $0 \le \theta \le \theta_{\max}$.

References

[1] R.M. Goodwin, *Econometrica* (1951) **19**, 1–17.
[2] R. H. Strotz, J.C. McAnulty, J.B. Naines, Jr. (1953) *Econometrica*, **21**, 390–411 .

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